Exo1

(1) • $A + A.B = A.(\underbrace{1+B}_{1}) = A.1 = \boxed{A}$

(2) • $A + \bar{A}.B = (\underbrace{A+\bar{A}}_{1}).(A+B)$

$= \boxed{A+B}$

(3) • $A(\bar{A}+B) = \underbrace{A.\bar{A}}_{0} + A.B = \boxed{A.B}$

(4) • $(A+B).(A+\bar{B}) = A.A + B.A + \cancel{B.\bar{B}} + A.\bar{B}$

$= A + A.B + A.\bar{B} + 0$

$= A.(\underbrace{1+B+\bar{B}}_{1}) = \boxed{A}$

• $(A+B+C).(A+B+\bar{C}) + A.B + A.C$

$(A+B+C).(A+B+\bar{C}) = A+B$

selon (4)

$A+B + A.B + A.C = A.(\underbrace{1+B+C}_{1}) + B$

$= \boxed{A+B}$

les lois de DEMORGAN

• $\overline{A+B} = \bar{A}.\bar{B}$

• $\overline{A.B} = \bar{A} + \bar{B}$

$\overline{(A+B)}.(\overline{\bar{A}+\bar{B}}) = 0$

$= \bar{A}.\bar{B}.(\bar{\bar{A}}.\bar{\bar{B}}) = \bar{A}.\bar{B}.A.B$

$= \underbrace{A.\bar{A}}_{0}.\underbrace{B.\bar{B}}_{0}$

(1) $= 0.0 = \boxed{0}$

• $\overline{A.\bar{B} + \bar{A}.B} = \overline{A.\bar{B}} . \overline{\bar{A}.B}$

$= (\bar{A} + \bar{\bar{B}}).(\bar{\bar{A}} + \bar{B})$

$= (\bar{A}+B).(A+\bar{B}) = \cancel{A.\bar{A}} + A.B + \cancel{B.\bar{B}} + \bar{A}.\bar{B}$

$= \boxed{A.B + \bar{A}.\bar{B}}$

• $(\bar{A}.\bar{B}).(A+\bar{A}.B) + \bar{C} + \bar{D} + \overline{C.D}$

$= \bar{A}.\bar{B}.A + \bar{A}.\bar{B}.\bar{A}.B + \overline{C.D} + \overline{C.D}$

$= \underbrace{\cancel{A.\bar{A}}}_{0}.\bar{B} + \bar{A}.\underbrace{\cancel{\bar{A}.B.\bar{B}}}_{0} + \overline{C.D} + \overline{C.D}$

$= \boxed{\overline{C.D}}$

Exo2

$F(A,B,C) = (A+\bar{B}).(\bar{A}+B+\bar{C})$

	A	B	C	$\bar{A}$	$\bar{B}$	$\bar{C}$	$A+\bar{B}$	$\bar{A}+B+\bar{C}$	F
0	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	0	1	1	1
2	0	1	0	1	0	1	0	1	0
3	0	1	1	1	0	0	0	1	0
4	1	0	0	0	1	1	1	1	1
5	1	0	1	0	1	0	1	0	0
6	1	1	0	0	0	1	1	1	1
7	1	1	1	0	0	0	1	1	1

Solution Série 3

FCD $\Rightarrow f(A,B,C) = \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C + A\cdot\bar{B}\cdot\bar{C} + A\cdot B\cdot\bar{C} + A\cdot B\cdot C$

FCC $\Rightarrow f(A,B,C) = (A+\bar{B}+C)\cdot(A+\bar{B}+\bar{C})\cdot(\bar{A}+B+\bar{C})$

FND $\Rightarrow f(A,B,C,D) = \sum(0,1,4,6,7)$

FNC $\Rightarrow f(A,B,C,D) = \prod(2,3,5)$.

Ex03

- $A.B.C + A\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot C + \bar{A}\cdot\bar{B}\cdot C + A\cdot\bar{B}\cdot\bar{C}$

$= A\cdot B\cdot(C+\bar{C}) + \bar{B}\cdot C\,(A+\bar{A}) + A\cdot\bar{B}\cdot\bar{C}$

$= A\cdot B + \bar{B}\cdot C + A\cdot\bar{B}\cdot\bar{C}$

$= A\cdot B + \bar{B}\cdot(C + A\cdot\bar{C}) = A\cdot B + \bar{B}\cdot((C+A)\cdot\underbrace{(C+\bar{C})}_{1})$

$= A\cdot B + \bar{B}\cdot(C+A) = A\cdot B + \bar{B}\cdot C + A\cdot\bar{B}$

$= A\cdot\underbrace{(B+\bar{B})}_{1} + \bar{B}\cdot C$

$= \boxed{A + \bar{B}\cdot C}$

- $\bar{A}\cdot\bar{B} + \overline{A+B+C+D}$ selon la loi de DE MORGAN

$\bar{A}\cdot\bar{B} + \overline{A+B+C+D} = \bar{A}\cdot\bar{B} + \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}$

$= \bar{A}\cdot\bar{B}\cdot\underbrace{(1 + \bar{C}\cdot\bar{D})}_{1}$

$= \boxed{\bar{A}\cdot\bar{B}}$

(2)

- $A\cdot B\cdot C + A\cdot B\cdot\bar{C} + A\cdot\bar{B}\cdot C\cdot D$

$= A\cdot B\underbrace{(C+\bar{C})}_{1} + A\cdot\bar{B}\cdot C\cdot D$

$= A\cdot B + A\cdot\bar{B}\cdot C\cdot D = A(B + \bar{B}\cdot C\cdot D)$

$= A\cdot(\underbrace{(B+\bar{B})}_{1}\cdot(B + C\cdot D))$

$= A(B + C\cdot D) = \boxed{A\cdot B + A\cdot C\cdot D}$

- $(A+B+C+D)\cdot(A+B+C+\bar{D})\cdot(\bar{A}+B+C)\cdot(B+\bar{C})$

sachant que $(X+Y)(X+\bar{Y}) = X$

Alors

$(A+B+C+D)\cdot(A+B+C+\bar{D}) = (A+B+C)$

$(A+B+C)\cdot(\bar{A}+B+C) = B+C$

$(B+C)\cdot(B+\bar{C}) = \boxed{B}$

- $\bar{A}\cdot\bar{B}\cdot\bar{C} + A\cdot B\cdot C + \bar{A}\cdot\bar{B}\cdot C + A B\bar{C} + A\cdot\bar{B}\cdot\bar{C} + A\cdot\bar{B}\cdot C$

$\Leftrightarrow \bar{A}\cdot\bar{B}\underbrace{(C+\bar{C})}_{1} + A\cdot B\underbrace{(C+\bar{C})}_{1} + A\cdot\bar{B}\cdot\underbrace{(C+\bar{C})}_{1}$

$= \bar{A}\cdot\bar{B} + A\cdot B + A\cdot\bar{B}$

$= \bar{B}\cdot\underbrace{(\bar{A}+A)}_{1} + A\cdot B = \bar{B} + A\cdot B$

$= (\bar{B}+A)\cdot\underbrace{(\bar{B}+B)}_{1}$

$= \boxed{\bar{B}+A}$

Solution Série 3

$$\bullet\ \underbrace{(A+B+C).(A+B+\bar{C})}_{} + A.B + B.C$$

$$(A+B) + A.B + B.C$$

$$= A + B + A.B + B.C$$

$$= A.\underbrace{(1+B)}_{1} + B.\underbrace{(1+C)}_{1}$$

$$= \boxed{A+B}$$

Exo 4

1)

$$F(A,B,C,D) = \sum(0,2,4,5,6,7,8,10,12,13,14,15)$$

$$= \prod(1,3,9,11)$$

	A	B	C	D	F(A,B,C,D)
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	0
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

2) La forme Abrégée est la forme Canonique Conjonctive FCC

$$F(A,B,C,D) = (A+B+C+\bar{D}).(A+B+\bar{C}+\bar{D})$$
$$.(\bar{A}+B+C+\bar{D}).(\bar{A}+B+\bar{C}+\bar{D})$$

3) Simplification, sachant que $(X+Y).(X+\bar{Y}) = X$ Alors

$\bullet\ (A+B+C+\bar{D}).(A+B+\bar{C}+\bar{D}) = A+B+\bar{D}$

$\bullet\ (\bar{A}+B+C+\bar{D}).(\bar{A}+B+\bar{C}+\bar{D}) = \bar{A}+B+\bar{D}$

Alors

$$F(A,B,C,D) = (A+B+\bar{D}).(\bar{A}+B+\bar{D})$$
$$= \boxed{B+\bar{D}}$$

• Simplification avec tableau de Karnaugh.

CD \ AB	00	01	11	10
00				
01	0			0
11	0			0
10				

Un groupe de 4 Zéros.

$$\boxed{F(A,B,C,D) = B + \bar{D}}$$

Solution Série 3

Exo 5

1) $F(A,B,C) = \bar{A}\cdot\bar{B}\cdot C + \bar{A}\cdot B\cdot\bar{C} + A\cdot B\cdot\bar{C}$

C \ AB	00	01	11	10
0		1	1	
1	1			

$= B\cdot\bar{C} + \bar{A}\cdot\bar{B}\cdot C$

2) $F(A,B,C) = A\cdot B + A\cdot\bar{B}\cdot C + \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot B\cdot C$

C \ AB	00	01	11	10
0	1		1	
1		1	1	1

$= \bar{A}\cdot\bar{B}\cdot\bar{C} + A\cdot B + B\cdot C + A\cdot C$

3) $F(A,B,C) = \sum(0,1,4,5)$

C \ AB	00	01	11	10
0	1			1
1	1			1

$= \bar{B}$

4)

CD \ AB	00	01	11	10
00		1		1
01		1		
11		1		
10		1		

$= \bar{A}\cdot B + A\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}$

5)

CD \ AB	00	01	11	10
00	1			
01	1	1		
11				
10	1	1		1

4 groupes de 2

$= \bar{A}\cdot\bar{B}\cdot\bar{C} + \bar{A}\cdot\bar{C}\cdot D + \bar{A}\cdot C\cdot\bar{D} + \bar{B}\cdot C\cdot\bar{D}$

6) $F(A,B,C,D) = \prod(1,2,3,5,7,8,9,12,14,15)$

$= \sum(0,4,6,10,11,13)$

CD \ AB	00	01	11	10
00	1	1		
01			1	
11				1
10		1		1

3 groupes de 2

+ 1 groupe 1

$F = A\cdot\bar{B}\cdot C + \bar{A}\cdot\bar{C}\cdot\bar{D} + \bar{A}\cdot B\cdot\bar{D} + A\cdot B\cdot\bar{C}\cdot D.$