Exercise Sheet 4

 # Boolean Algebra

Exercise 1

1. Demonstrate De Morgan's laws using Boolean algebra theorems.
2. Using De Morgan's laws, prove the following equalities

1) $\overline{A}+\overline{D}+(B+A.D)+(C+D)=1$ 2) $A.\overline{B}+C+(\overline{A}+B).\overline{C}=1$ 3) $A.B+B.C+\overline{A}.B.\overline{C}=B$

4) $(\overline{A+B}).(\overline{\overline{A}+\overline{B}})=0$ 5) $\overline{A.\overline{B}+\overline{A}.B}=A.B+\overline{A}.\overline{B}$

Exercise 2

1. Find the dual and the complement of the following Boolean functions

$F1=\overline{A}.B.\overline{C}+\overline{A}.\overline{B}.C$ $F2=A.(\overline{B}.\overline{C}+B.C)$ $F3=(\overline{A+B}).(C+D)$

Exercise 3

Prove the following equalities using Boolean algebra theorems

1) $A+A.B=A$ 2) $A+\overline{A}.B=A+B$ 3) $A.(\overline{A}+B)=A.B$ 4) $(A+B).(A+\overline{B})=A$

5) $(A+B+C).(A+B+\overline{C})+A.B+A.C=A+B$ 6) $A.B+A.C.D+\overline{B}.D=A.B+\overline{B}.D$

7) $A.B+A.\overline{B}.C=A.B+A.C$ 8) $(A+B).(\overline{A}+C).(B+C)=(A+B).(\overline{A}+C)$ 9) $A.\overline{B}+C+(\overline{A}+B).\overline{C}=1$

Exercise 4

Reduce the following Boolean expressions using Algebraic simplification

1) $(A.B+A.\overline{B}.\overline{C}+B.C+\overline{A}.B.\overline{C}+A.C+\overline{A}.\overline{B}.C).\overline{C}$ 2) $\overline{A}.B+\overline{A}.B.\overline{C}+\overline{A}.B.C.D+\overline{A}.B.\overline{C}.\overline{D}.E$

3) $\overline{A}.\overline{B}.\overline{C}.\overline{D}+A.\overline{B}.\overline{C}.\overline{D}+\overline{A}.\overline{B}.C.\overline{D}+A.\overline{B}.C.\overline{D}$ 4) $(A+B+C)(A+B+\overline{C})+A.B+B.C$

5) $\overline{A}.C+B.D+A.C.\overline{D}+\overline{B}.D$ 6) $A.(A+B+C).(\overline{A}+B+C)(A+\overline{B}+C).(A+B+\overline{C})$

7) $A.B+B.C+A.C+A.\overline{B}.\overline{C}+\overline{A}.B.\overline{C}+\overline{A}.\overline{B}.C$ 8) $A.B.C+A.\overline{B}.\overline{C}$ 9) $\overline{A}.\overline{B}+\overline{A+B+C+D}$

Exercise 5

Convert F1 and F2 functions to the first canonical form, F3 and F4 to the second canonical form.

$F1(A,B)=\overline{B}+A$ $F2(A,B,C)=A.B+\overline{B}.C+\overline{C}$

$F3(A,B)=\overline{A}$ $F4(A,B,C)=(\overline{B}+A).(A+\overline{C})$

Exercise 6

Reduce the following Boolean functions to a minimum number of operators

$$F(A,B,C,D)=\overline{A}.\overline{B}.\overline{C}.\overline{D}+\overline{A}.B.\overline{C}.D+A.B.C.D+A.\overline{B}.C.\overline{D}$$
$$G(A,B,C,D)=A.\overline{B}.\overline{C}.\overline{D}+A.B.\overline{C}.D+\overline{A}.B.C.D+\overline{A}.\overline{B}.C.\overline{D}$$
$$H(A,B,C,D)=F(A,B,C,D)+G(A,B,C,D)$$
$$K(A,B,C,D)=F(A,B,C,D).G(A,B,C,D)$$

Exercise 7

Express the Boolean function F in DCF, CCF and numerical forms. $F(A,B,C)=\overline{A.(B.C+\overline{C})+\overline{B}}$

Exercise 8

Let *F(A,B,C)* a Boolean function.

$$F(A,B,C) = \sum(0,2,4,6,7)$$

1- Use the Karnaugh map to simplify F (using minterms/maxterms)
2- Draw the logic diagram only with NAND gates
3- Draw the logic diagram only with NOR gates and inverters
4- Draw the logic diagram only with NOR gates

Exercise 9

Using the Karnaugh map, minimize the following functions

1)$F(A,B,C) = \overline{A}.\overline{B}.C + \overline{A}.B.\overline{C} + A.B.\overline{C}$

2)$F(A,B,C) = A.B + A.\overline{B}.C + \overline{A}.\overline{B}.\overline{C} + \overline{A}.B.C$

3)$F(A,B,C) = \sum(0,1,4,5)$

4)$F(A,B,C) = (A+B+C).(A+B+\overline{C}).(\overline{A}+B+C).(\overline{A}+B+\overline{C})$

5)$F(A,B,C) = \overline{(A.\overline{B}+A.\overline{C}).\overline{B.C}}$

6)$F(A,B,C,D) = \overline{A}.B.\overline{C}.\overline{D} + \overline{A}.B.\overline{C}.D + \overline{A}.B.C.D + \overline{A}.B.C.\overline{D} + A.\overline{B}.\overline{C}.\overline{D}$

7)$F(A,B,C,D) = \overline{A}.\overline{B}.\overline{C} + \overline{A}.\overline{B}.C.\overline{D} + A.\overline{B}.C.\overline{D} + \overline{A}.B.C.\overline{D} + \overline{A}.B.\overline{C}.D$

8)$F(A,B,C,D) = \prod(1,2,3,5,7,8,9,12,14,15)$

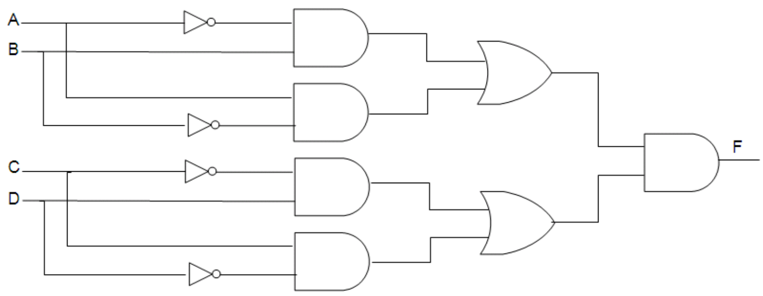
9)$F(A,B,C,D,E) = \sum(3,13,15,19,21,23,29,31)$

10)$F(A,B,C,D,E) = \sum(3,4,6,13,15,19,20,22,29,31)$

Exercise 10

Given the logic diagram of the function F.

1- Find the Boolean expression of F
2- Reduce algebraically F and draw the equivalent logic diagram.
3- What are the benefits of the new circuit?



Exercise 11

Show only the groupings made in the following K-maps

CD \ AB	00	01	11	10
00	1	1		1
01				1
11	1			
10	1		1	1

CD \ AB	00	01	11	10
00	1			
01	1		X	
11	1			
10	X			

CD \ AB	00	01	11	10
00	1	1	1	
01	1	1	1	
11		1	X	
10	X			

CD \ AB	00	01	11	10
00	1	X		
01				X
11				X
10	1	X		

CD \ AB	00	01	11	10
00	1			
01		1		
11			1	
10				1

CD \ AB	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			X

CD \ AB	00	01	11	10
00	1	1		
01		1	1	
11			1	1
10				1

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1			1
10	1	1	1	1

CD \ AB	00	01	11	10
00	1	1	1	1
01		1		
11	1			1
10			1	

CD \ AB	00	01	11	10
00	1	X		
01	X	X		
11			1	1
10			X	